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I SEMESTER BCA(AI) EXAMINATION - NOVEMBER/DECEMBER 2025

SCHEME: Revised CBCS

041

BCA (Artificial Intelligence)
Linear Algebra

Time: 03 Hours **Max Marks: 80**

Instruction: Answer both **Part A** & **Part B.**

PART - A

1. Answer **ALL** questions. **8x2=16**

   a. Define Field. CO1 LL1

   b. Define LU Factorization. CO1 LL1

   c. What is Dimension of Vector Space? CO2 LL1

   d. What is Vector Space? CO2 LL1

   e. Define Linear Functional. CO3 LL1

   f. Define the Length (or Norn). CO3 LL1

   g. What is Symmetric Matrix? CO4 LL1

   h. Define Singular Valued Decomposition. CO4 LL1

PART - B

Answer any **TWO** sub questions from each main. **4x16=64**

2. a) Compute each matrix sum or product if it is defined, if an expression is undefined, explain why? Let CO1 LL2 8

$A=\begin{bmatrix}2 & 0 & -1\\ 4 & -5 & 2\end{bmatrix}$, $B=\begin{bmatrix}7 & -5 & 1\\ 1 & -4 & -3\end{bmatrix}$, $C=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}$, $D=\begin{bmatrix}3 & 5\\ -1 & 4\end{bmatrix}$, $E=\begin{bmatrix}-5\\ 3\end{bmatrix}$

i) -2A ii) B-2A iii) AC iv) CD

b) Let $A=\begin{bmatrix}1 & -3\\ -2 & 4\end{bmatrix}$, $X=\begin{bmatrix}5\\ 3\end{bmatrix}$ Compute $(AX)^T$, X^TA^T, XX^T, X^TX. Is A^TX^T defined?

CO1 LL2 8

c) Find the general solution of the linear system whose augmented matrix has been reduced to CO1 LL2 8

$$\begin{bmatrix}1 & 6 & 2 & -5 & -2 & -4\\ 0 & 0 & 2 & -8 & -1 & 3\\ 0 & 0 & 0 & 0 & 1 & 7\end{bmatrix}$$

PTO

d) Determine the existence and uniqueness of the solutions to the system. CO1 LL2 8

$$3x_2 - 6x_3 + 6x_4 - 4x_5 = -5$$
$$3x_1 - 7x_2 + 8x_3 - 5x_4 + 8x_5 = 9$$
$$3x_1 - 9x_2 + 12x_3 - 9x_4 + 6x_5 = 15$$

3. a) Let $B=\{b_1,b_2,\ldots.b_n\}$ be a basis for a vector space V. Then prove that the co-ordinate mapping $x\rightarrow[x]_B$ is one to one linear transformation from V onto R^n. CO2 LL2 8

b) Let $S=\{1,t,t^2,\ldots\ldots t^n\}$, verify that S is a basis for P_n. This basis is called the standard basis for P_n. CO2 LL2 8

c) i) Prove that the Row-equivalent matrices have the same row space. CO2 LL2 8

ii) Write a note on :

a) Basis of a Subspace. b) Isomorphism

d) Let V,W,Z be vector space over the field F. Let T be a linear transformation from V into W and U is a linear transformation from W into Z. then prove that the composed function UT define by $(UT)(\alpha)=U(T(\alpha))$ is a Linear Transformation form V into Z.

CO2 LL2 8

4. a) Prove that: Let F be a field and let T be the operator on F^2 defined by $T(x_1, x_2) = (x_1, 0)$

CO3 LL2 8

b) i) Let $v = (1,-2,2,0)$, Find the unit vector u in the same direction as v. CO3 LL2 4

ii) Compute $u - v$ and $v - u$ for $u = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}$ CO3 LL2 4

c) Show that $L(g) = \int_a^b g(t)dt$, defines a linear functional on the space C([a,b]) of continuous real-valued functions on [a, b]. CO3 LL2 8

d) Let n be a positive integer and F a field. If A is an n x n matrix with entries in F, define the trace of A as $t_r(A)=a_{11}+a_{22}+\ldots..+a_{nn}$, the sum of the diagonal entries of A. Show that the trace function is a linear functional on the F^{nxn}.

CO3 LL2 8

PTO

5. a) Let $x=\begin{bmatrix}x_1\\x_2\end{bmatrix}$. Compute x^TAx for the following matrices: CO4 LL2 8

i) $A=\begin{bmatrix}4 & 0\\0 & 3\end{bmatrix}$ ii) $A=\begin{bmatrix}3 & -2\\-2 & 7\end{bmatrix}$

b) Find the maximum and minimum values of $Q(x)=9x_1^2+4x_2^2+3x_3^2$ subject to the constraint $x^Tx=1$. CO4 LL2 8

c) Prove the following:

Let A be an n x n symmetric matrix. Then a quadratic form x^TAx is:

i. positive definite if and only if the eigenvalues of A are all positive.
ii. negative definite if and only if the eigenvalues of A are all negative.
iii. indefinite if and only if A has both positive and negative eigenvalues.

d) State and Prove QR Factorization. CO4 LL2 8

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